A simple analysis of Expressnet

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UNE ANALYSE SIMPLE D'EXPRESSNET

Abstract :

The Expressnet protocol works on a unidirectional transmission medium. Users, physically ordered, can access the medium according to a distributed conflict-free round-robin algorithm. In this paper, we examine Expressnet performance when the most upstream station is in charge of generating trains of packets.The analysis in the case of symmetric Poisson traffic is based on a Markov model. We derive a simple formula for the mean access-delay and we compare it with results of simulations.

Resumé:

Le protocole Expressnet opère sur un médium de communication unidirectionel. Les utilisateurs physiquement ordonnés sur le médium reçoivent des droits d'accès suivant un algorithme sans conflit de type round-robin. Dans ce papier, nous examinons le cas où la station la plus en amont se charge de générer les trains de paquets. L'analyse dans le cas d'un trafic de Poisson symétrique est basée sur un modèle de Markov. Nous en déduisons une formule simple pour le délai d'accès moyen et nous comparons les résultats du modèle analytique et ceux de la simulation.

I. Introduction :

With the increasing need for sophisticated communications and the scaling up of computer speed, high speed communications will become more and more mandatory .

Research work in this area is primarily devoted to the design of communication channels whose capacity is in the order of 1 Gbit/s . Such a capacity calls for a new generation of communication . Existing protocols (e.g Ethernet or the Token passing bus) are obviously not efficient for high speed local area networks .

Expressnet has been one of the first protocols designed for high speed local area networks . It operates on a special topology where the stations are physically ordered [1] . This protocol has been analyzed in the case of a single buffer with loss [2].Our aim here is to give a simple analysis of this protocol with the classical hypothesis: symmetric Poisson traffic and infinite buffer . We will moreover suppose that the most upstream station is in charge of generating trains. This paper is divided into three sections. Section II is a very brief description of Expressnet . In section III we introduce our Markov model, which is used to express the distribution of the number of stations in a train with respect to the total input load and the propagation delay. In section IV we derive a simple queueing model which leads us to a formula for the mean access delay . Section V compares the theoretical model with results of simulations . In the appendix one can find all the intermediate formulas which lead to the expression of the mean length of the queue.

II. Expressnet as a unidirectional broadcast system :

Expressnet is based on a unidirectional broadcast medium consisting of one outbound channel connected to one inbound channel. Fig.1 gives one possible topology for Expressnet. Each station writes on the outbound channel . Packet are read by the addressees on the inbound channel. The main idea behind this protocol is to create trains of messages : each station having a pending packet senses the outbound channel to find the end of a train and then transmits immediately . Doing so increases the length of the train . A train can include at most N packets where N is the number of active stations . With the protocol analyzed in the following, the most upstream station is in charge of generating new trains. When this very station senses the end of a train on the inbound channel it starts a new train, with the transmission of a packet if one is waiting, with a burst which is long enough to be sensed on the outbound channel otherwise. Fig.2 helps to understand how this protocol works.

In fact, due to delays between sensing the begin-of-carrier and aborting transmission, every packets must include a bumper . For details, a complete description of Expressnet is given in [1] . The protocol analyzed in this paper slightly differs from Expressnet where a " cold start " procedure may be used to generate trains, rather than relying on a particular station . However, as far as access delays are concerned, Expressnet with the most upstream station generating trains is better than the complete version of Expressnet .

III. A model of Expressnet with the most upstream station generating trains:

N is the number of active stations.
Let 1 be the time duration of a packet.
Let Y be the propagation delay between the outbound tap and the inbound tap of a user (Y is user independent) .

Let λ be the total input load.

The duration of the burst generated by the most upstream station is supposed to be negligible compared to 1 and Y.

First of all we can notice that distances between stations are not significant parameters in the case of Poisson traffic. We can adjust the local time in every station so that a burst propagating on the channel is detected at "the same time"on the outbound channel by every station . Thus changing distances is equivalent to changing local times which does not modify the Poisson properties of traffic.
Another very peculiar thing is the fact that the mean access delay is station independent . For the most upstream station the time interval between two successive access rights is equal to Y plus the duration of the train . For any other station, the time interval between two successive access rights can be decomposed into the three following elements : the time interval between the first access right and the end of the current train, the propagation delay Y and the time interval between the beginning of the next train and the following access right. The sum of these three elements has the same mean as the average access delay for the most upstream station.
We conjecture that the delay distribution between two access rights is almost independent on the stations. Simulations in [2] seem to reenforce this assertion. Our simple model is based on this hypothesis.
Thus, for a given station, we can define a train as the packets generated on the network between two access rights .The state of our Markov chain will simply be the number M of packets in the train.The time between two consecutive access rights is : $L = Y + M$, which we will call a cycle .

With a total input load λ, our model assumes that the transition probability from state M to state M' is:

$$P(M'/M) = \frac{\lambda^{M'}\left(Y+M\right)^{M'}}{M'!} e^{-\lambda\left(Y+M\right)} \quad . \tag{1}$$

Let g_M denotes the probability for a cycle to include M packets. The characteristic function of the number of packets generated in a cycle is :

$$g(z) = \sum_{M \geq o} g_M z^M .$$

The equation of the steady state is :

$$g(z) = \sum_{M \geq o,\, M' \geq 0} g_M P(M' \mid M)\ z^{M'} .$$

Using formula (1),we have:

$$g(z) = \sum_{M \geq o,\, M' \geq 0} g_M \frac{\lambda^{M'}\left(Y+M\right)^{M'}}{M'!} e^{-\lambda(Y+M)} z^{M'} ,$$

$$g(z) = \sum_{M \geq o} g_M \, e^{\lambda(Y+M)(z-1)} ,$$

$$g(z) = e^{\lambda Y(z-1)} \, g(e^{\lambda(z-1)}) . \qquad (2)$$

This equation provides us with the information we need about the distribution of the number of packets in a train.For example :

$$g'(1) = \frac{\lambda Y}{1-\lambda} \qquad (Cf \text{ Appendix}).$$

$g'(1)$ is the mean number of packets in a train denoted by $E(M)$.

$$E(M) = \frac{\lambda Y}{1-\lambda} \qquad or \qquad \lambda = \frac{E(M)}{E(M)+Y} .$$

The mean duration of a train is :

$$E(L) = Y + E(M) = \frac{Y}{1-\lambda} . \qquad (3)$$

The variance of a train duration is :

$$Var(L) = \frac{\lambda Y}{(1-\lambda)(1-\lambda^2)} \qquad (Cf \text{ Appendix}) . \qquad (4)$$

IV. The queueing model

For a given station let q_n denote the probability for the queue to be of length n just before an access right. The generating function of the queue is:

$$q(z) = \sum_{n \geq o} q_n z^n .$$

The transition scheme is :

$n \neq 0 \qquad n \longrightarrow n-1+A$

$n = 0 \qquad n \longrightarrow A$

where A is a Poisson arrival process of rate λ/N during a cycle. The duration of the cycle is

$L = M + Y$. We suppose that the cycle duration and the queue length are independent processes. Let $a(z)$ be the characteristic function of A. The steady state is given by the following equation [4] :

$$\left(\frac{q(z) - q_o}{z} + q_o \right) a(z) = q(z) \quad .$$

We have:

$$q(z) = q_o \frac{1 - z}{1 - \frac{z}{a(z)}} \quad . \tag{5}$$

with q_0 defined by the condition $q(1) = 1$.(Cf appendix).
$a(z)$ is obviously given by the following formula:

$$a(z) = \sum_{M \geq o} g_M \, e^{\frac{\lambda (M + Y)(z-1)}{N}} \quad .$$

Thus, we obtain :

$$a(z) = e^{\frac{\lambda Y(z-1)}{N}} \, g\left(e^{\frac{\lambda (z-1)}{N}} \right) \quad . \tag{6}$$

As we know $g(z)$ we know $a(z)$ and $q(z)$. We are now interested in deriving the mean access delay noted W. The mean number of complete waiting cycles exprienced by a random customer is given by Little formula.

$$W = \frac{E(q)}{1 - q_o} - 1 \, .$$

Thus the mean waiting experienced by a random customer is [4]:

$$W = \left(\frac{E(q)}{1 - q_o} - 1 \right) E(L) + \frac{D(L)}{2} \quad . \tag{7}$$

where $D(L)$ is the mean duration of the first cycle during which the customer arrives in queue [4]:

$$D(L) = E(L) + \frac{Var(L)}{E(L)} \, .$$

Thus :

$$W = \left(\frac{E(q)}{1-q_o} - \frac{1}{2} \right) E(L) + \frac{1}{2} \frac{Var(L)}{E(L)} .$$

Finally we find :

$$W = \frac{Y}{2(1-\lambda)\left(1 - \frac{\lambda Y}{N(1-\lambda)}\right)} \left(1 + \frac{\lambda^2}{N(1-\lambda^2)}\right) + \frac{1}{2} \frac{\lambda}{1-\lambda^2} .$$

W tends to ∞ as λ tends to λ_{max} where λ_{max} is :

$$\lambda_{max} = \frac{N}{N+Y}$$

λ_{max} is the maximum capacity for Expressnet with N active stations.That is is a known result.

V. Comparaison between the results of the analytical model and of simulations.

We have tested our model with $Y = 20$, $N = 50$ and $Y = 4$,$N = 10$. We have simulated Expressnet (the most upstream station generating trains) with symmetric Poisson traffic and identical packet duration taken as time unit. Simulation software has been written in C++ under Sphinx. Sphinx is a performance evaluation tool developed at INRIA for the analysis of local area networks [3] .
Curves of the mean access delay versus the input load shown on figures 3 and 4 demonstrate the accuracy of our simple model, especially when the parameter values are large.
On figures 5, 6 and 7, results related to the variance of a train duration obtained out of our analytical model are compared with simulation results. The matching is quite good, especially when N is large .

VI. Conclusion

We have investigated the performance of a basic version of Expressnet. To this purpose we have developed a simple and robust model which seems well adapted to solving the problem.The complete version of Expressnet includes a "cold start " procedure which precludes the need for relying on a particular station for generating trains. The same model with some additions could be used to evaluate the complete version of Expressnet. Nevertheless the version evaluated in this paper is better as far as access delays are concerned.

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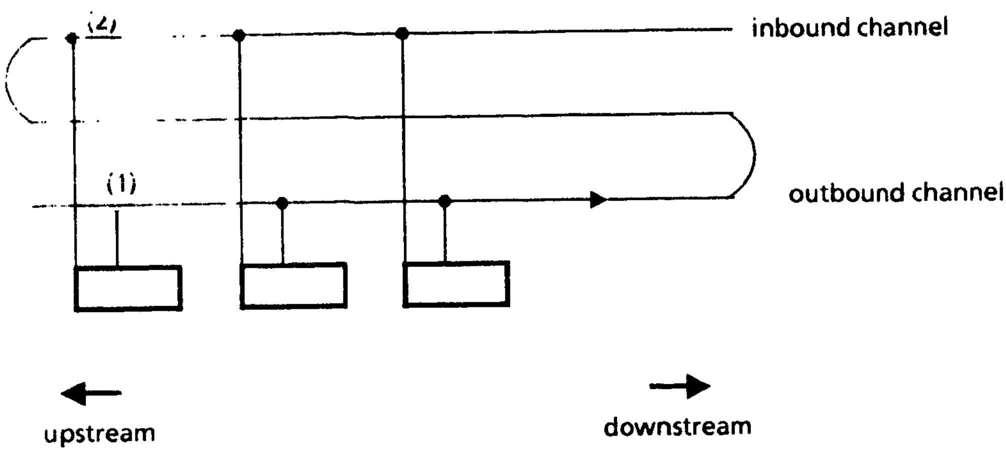
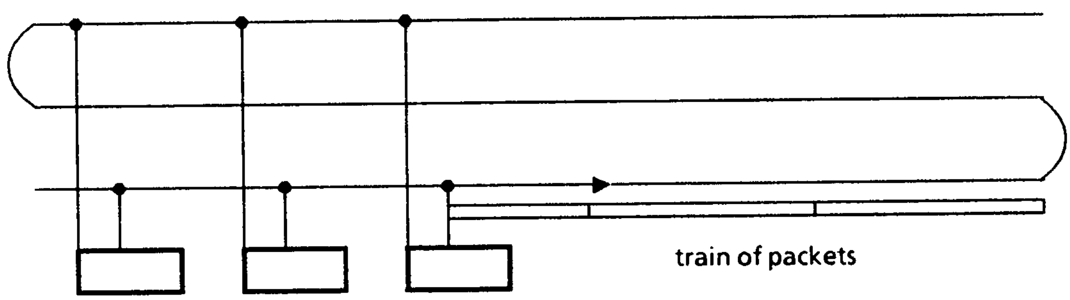


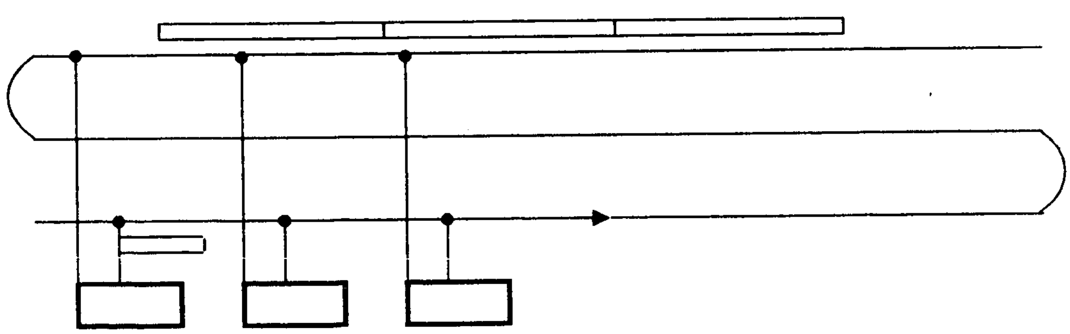
(1) outbound tap of the most upstream station

(2) inbound tap of the most upstream station

Figure 1



a station adds a packet to the train



the most upstream station senses the end of the train and starts transmitting

Figure 2

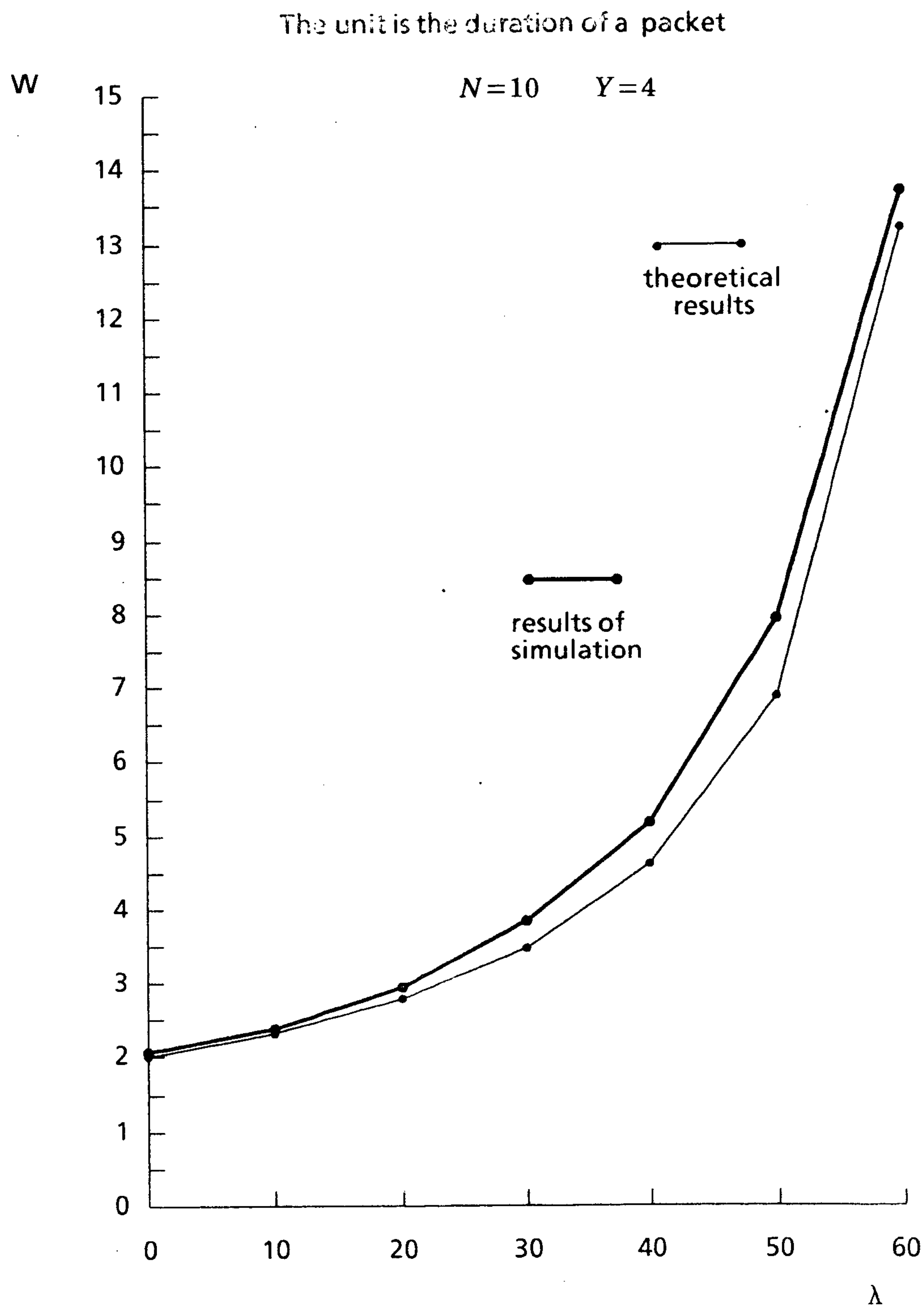


Figure 3 : The mean access delay versus the input load in percent

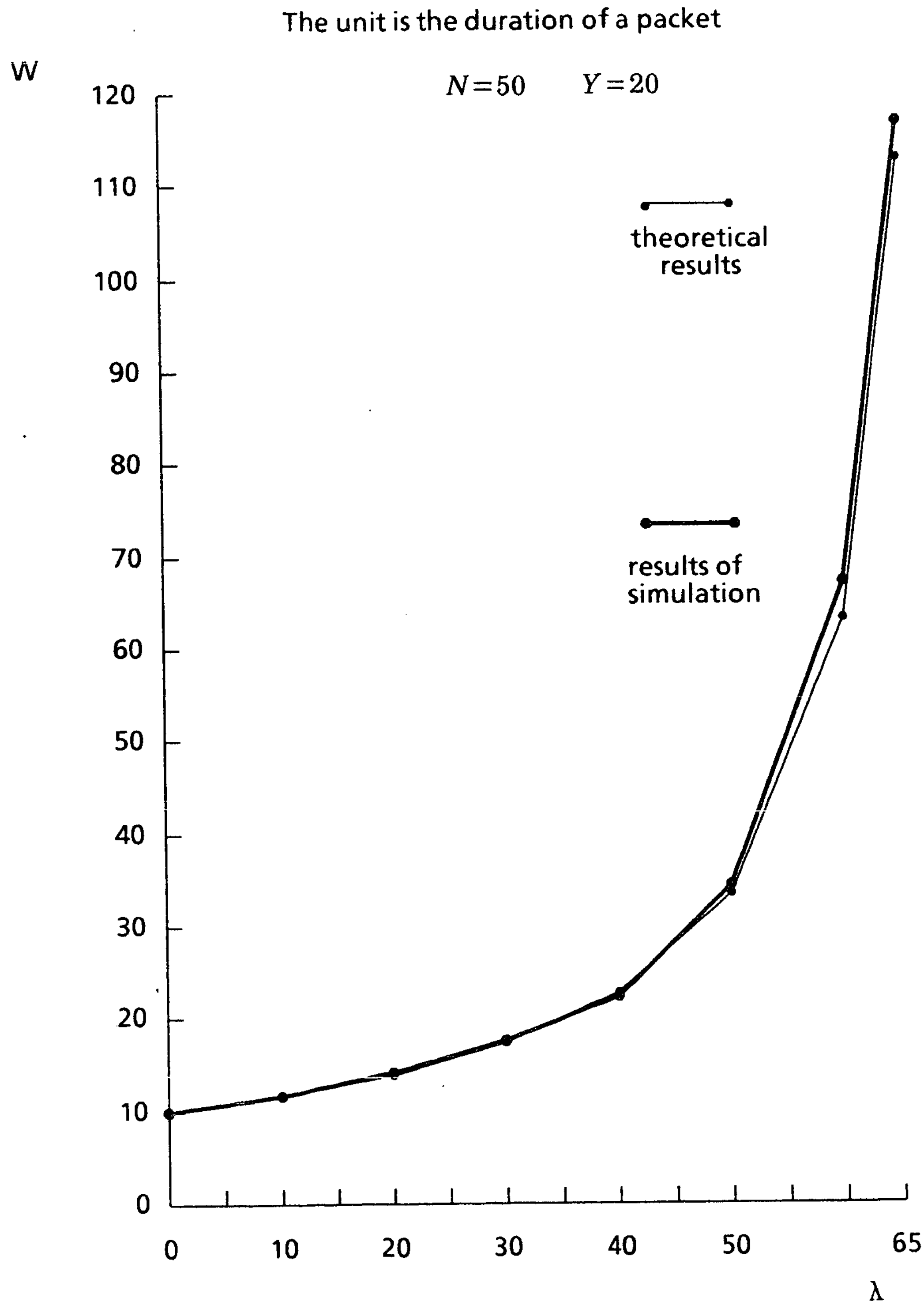


Figure 4 : The mean access delay versus the input load in percent

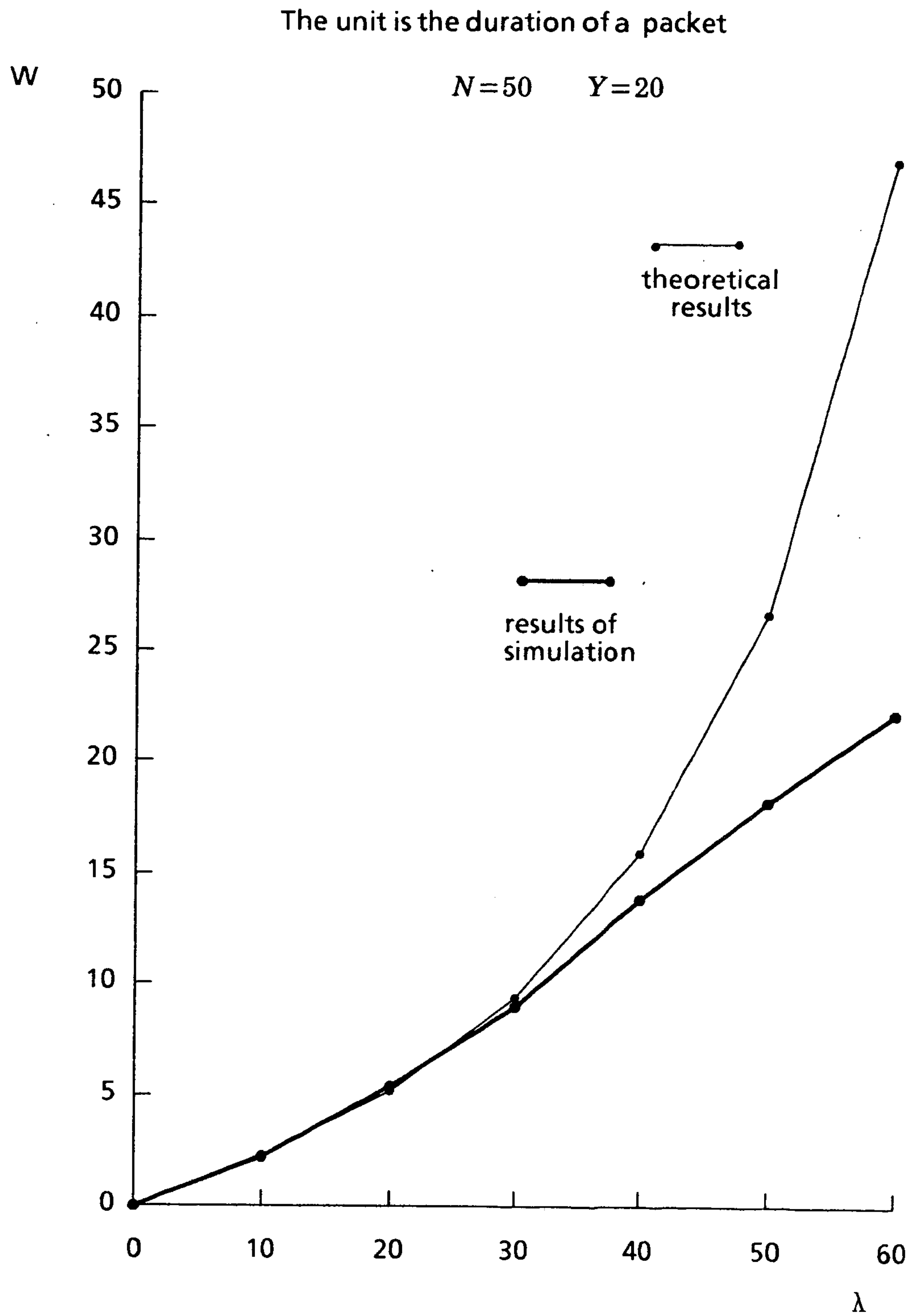


Fig 5 : Variance of a train duration versus input load in percent

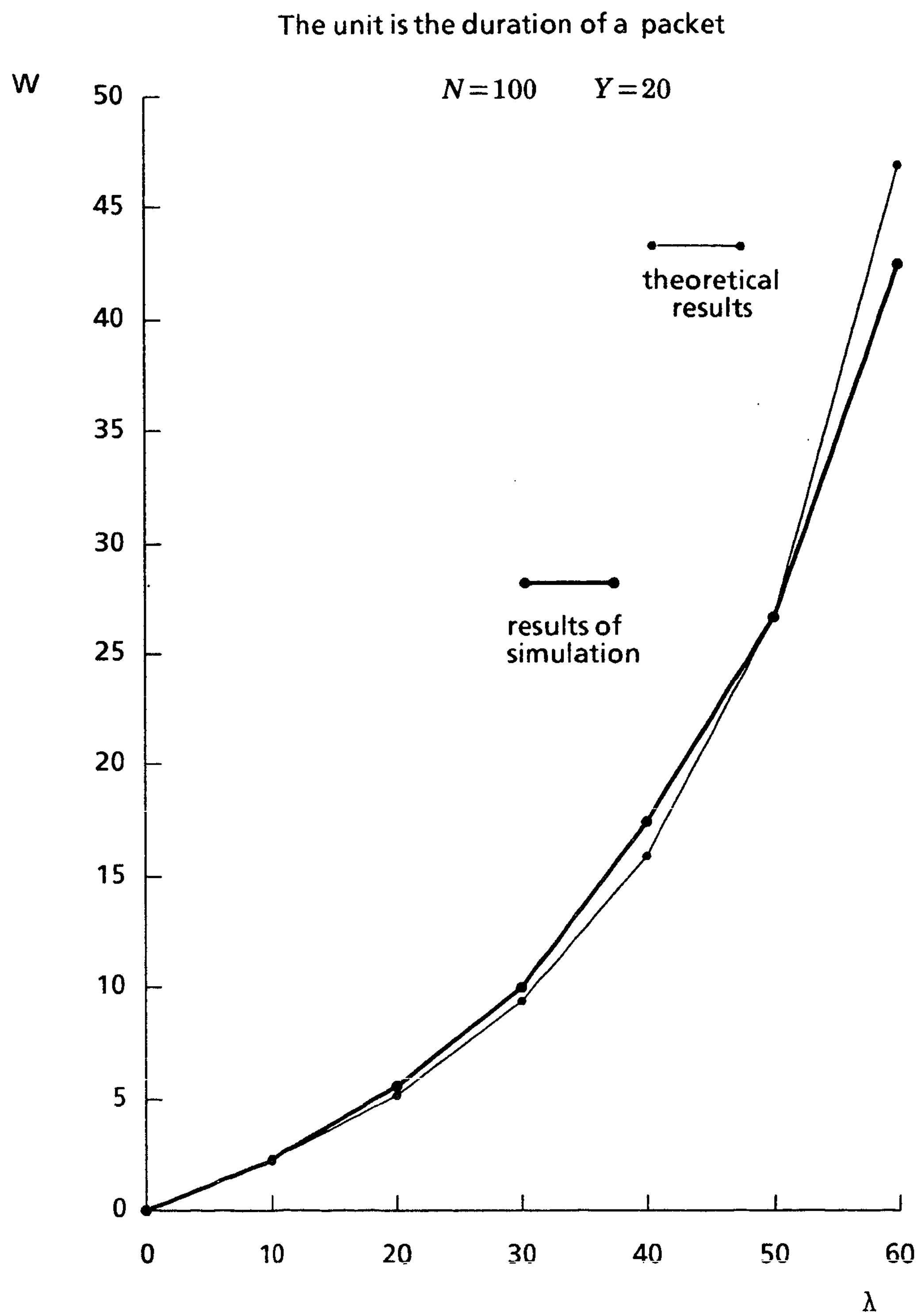


Fig 6 : Variance of a train duration versus input load in percent

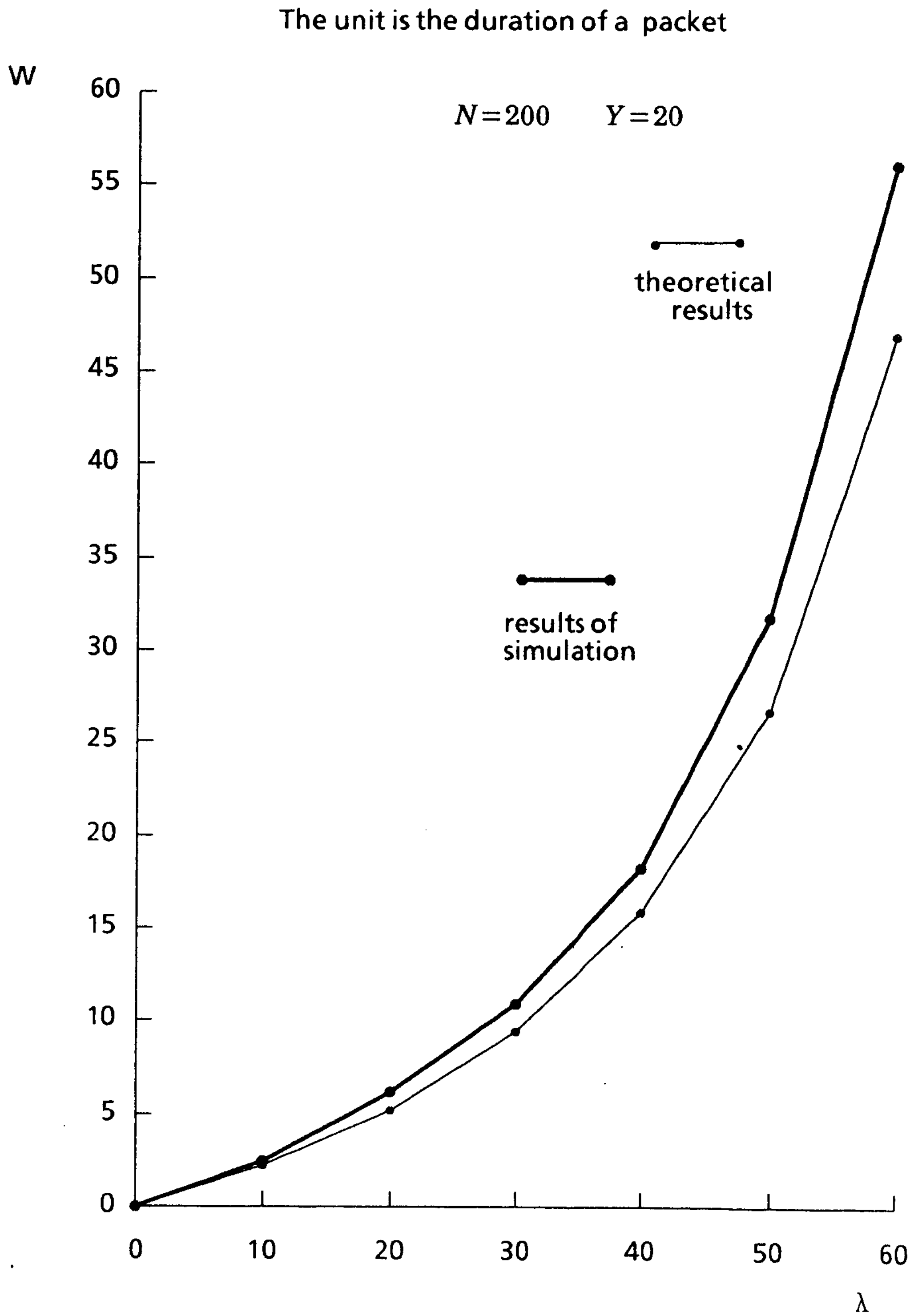


Fig 7 : Variance of a train duration versus input load in percent

Appendix

A Formulas for $g'(1), g''(1)$:

Deriving (1) we get:

$$g'(z)=\lambda Y e^{\lambda Y(z-1)} g(e^{\lambda(z-1)})+\lambda e^{\lambda(Y+1)(z-1)} g'(e^{\lambda(z-1)}) . \qquad (8)$$

For $z = 1$ we have:

$$g'(1)=\lambda Y g(1)+\lambda g'(1) \qquad g(1)=1 ,$$

$$g'(1)=\frac{\lambda Y}{1-\lambda} .$$

Deriving (8) we get:

$$g''(z)=\lambda^2 Y^2 e^{\lambda Y(z-1)} g(e^{\lambda(z-1)})+\lambda^2(2Y+1) e^{\lambda(Y+1)(z-1)} g'(e^{\lambda(z-1)})$$

$$+\lambda^2 e^{\lambda(Y+2)(z-1)} g''(e^{\lambda(z-1)}).$$

For $z = 1$ we have:

$$g''(1)=\lambda^2 \frac{Y+\lambda Y+\lambda}{(1-\lambda)(1-\lambda^2)} ,$$

$$Var(L)= Var(M) ,$$

$$Var(M)=\sum_{M\geq 0} M^2 g_M-(E(M))^2 = g''(1)-(g'(1))^2+g'(1) .$$

Finally we find:

$$Var(L) = \frac{\lambda Y}{(1-\lambda)(1-\lambda^2)} .$$

B Expansion of $a(z)$ in the neighbourhood of $z = 1$

Equation (6) reads as follows :

$$a(z) = e^{\frac{\lambda Y(z-1)}{N}} \quad g\left(e^{\frac{\lambda(z-1)}{N}} \right) .$$

Therefore :

$$a(z) = \left(1 + \frac{Y}{N}(z-1) + \frac{Y^2}{N^2}\frac{(z-1)^2}{2} + O((z-1)^3)\right) g\left(e^{\frac{\lambda(z-1)}{N}} \right) .$$

Let $b(z)$ denote

$$b(z) = g\left(e^{\frac{\lambda(z-1)}{N}} \right) ,$$

$$b'(1) = g'(1) ,$$

$$b''(1) = \frac{\lambda^2}{N^2}\left(g'(1) + g''(1) \right) ,$$

$$b(z) = 1 + \frac{\lambda}{N} g'(1)(z-1) + \frac{\lambda^2}{N^2}\left(g'(1) + g''(1) \right)\frac{(z-1)^2}{2} + O((z-1)^3) ,$$

$$a(z) = 1 + \frac{\lambda}{N} E(L)(z-1) + \frac{\lambda^2}{N^2}(Y^2 + 2Y g'(1) + g'(1) + g''(1)) \frac{(z-1)^2}{2} + O((z-1)^3) .$$

Let us define K as:

$$K = Y^2 + 2Y g'(1) + g'(1) + g''(1) . \qquad (9)$$

Thus:

$$a(z) = 1 + \frac{\lambda}{N} E(L)(z-1) + \frac{\lambda^2}{N^2} K \frac{(z-1)^2}{2} + O((z-1)^3) . \qquad (10)$$

C Computations of q_0 and $E(q) = q'(1)$

Expansion of $a(z)$ can now be used to get q_0 and $E(q)=q'(1)$. Using (5) and (10) at the first order we have:

$$q(z) = q_0 \frac{(1-z)\left(1+\frac{\lambda}{N}E(L)(z-1)+O((z-1)^2)\right)}{1-z+\frac{\lambda}{N}E(L)(z-1)+O((z-1)^2)} .$$

When z tends to 0 we obtain:

$$q(1) = q_0 \frac{1}{\left(1-\frac{\lambda}{N}E(L)\right)} = 1 ,$$

or

$$q_0 = 1 - \frac{\lambda}{N}E(L) . \qquad (11)$$

The expansion of $a(z)$ at the second order can be used to compute $q'(1)$ as follows :

$$\Delta q(u) = \frac{q(z)-1}{z-1} \quad \text{with } u=z-1 .$$

Eq. (5) allows us to calculate $\Delta q(u)$ as a function of u and $a(u+1)$:

$$\Delta q(u) = \frac{a(u+1)-(u+1)+q_0 u\, a(u+1)}{u\left(a(u+1)+u+1\right)} ,$$

$$\Delta q(u) = \frac{-u\left(1-\frac{\lambda}{N}E(L)-\frac{\lambda^2}{N^2}K\frac{u}{2}\right)+u\left(1-\frac{\lambda}{N}E(L)\right)\left(1+\frac{\lambda}{N}E(L)u+\frac{\lambda^2}{N^2}K\frac{u^2}{2}\right)+O(u^3)}{-u\left(-u+\frac{\lambda}{N}E(L)u\right)+O(u^3)} .$$

$\Delta q(u)$ tends to $q'(1)$ as u tends to 0,

$$E(q) = q'(1) = \frac{\frac{\lambda}{N}E(L)+\frac{K}{2}\frac{\lambda^2}{N^2}-\frac{\lambda^2}{N^2}E(L)^2}{1-\frac{\lambda}{N}E(L)} . \qquad (12)$$

D Computation of the mean access delay: W

Using formulas (7),(11), (12) we get:

$$W = E(L)\ \frac{\frac{1}{2} + \frac{\lambda}{2N}\left(\frac{K}{E(L)} - E(L)\right)}{1 - \frac{\lambda}{N} E(L)}\ . \tag{13}$$

Finally, using Eq (3), (4), (9), (13) together we obtain:

$$W = \frac{Y}{2(1-\lambda)\left(1 - \frac{\lambda Y}{N(1-\lambda)}\right)}\left(1 + \frac{\lambda^2}{N(1-\lambda^2)}\right) + \frac{1}{2}\ \frac{\lambda}{1-\lambda^2}\ .$$

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